Inertial Attitude Determination for a Dual-Spin Planetary Spacecraft

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Design and performance evaluations of an onboard inertial attitude determination system for the dual-spin Galileo spacecraft are presented. The Galileo spacecraft is designed for a planetary mission to Jupiter. A quaternion integration algorithm and a rate estimator sequentially determine the scan platform's inertial attitude and rate from the drift and misalignment compensated gyro outputs. A least-squares estimator algorithm processes the star transit data from a rotor-mounted star scanner, and provides periodic updates of the scan platform's attitude as a means of correcting the gyro drift and the error drift of the quaternion integration algorithm. Computer simulated algorithm performance in the presence of nutation and sensor noise are presented.

I. Introduction

THE Galileo project is a planetary mission to Jupiter scheduled for launch in 1986. This mission's main objective is to conduct intensive investigation of the Jovian atmosphere, satellites, and magnetosphere. The project involves the development of a dual-spin spacecraft for orbiting the planet, and a probe, to be released prior to Jupiter orbit insertion, for investigating the planet's atmosphere.[1,2] This planetary mission requires spacecraft attitude determination based on autonomous capabilities because of significant mission complexity and the immense distance from Earth. The spacecraft carries all the necessary celestial and inertial sensors, as well as onboard software algorithms stored in a digital computer to generate its own attitude reference in real time. Three types of sensors are used for determining the attitude of the dual-spin spacecraft: A dry inertial reference unit (DRIRU) to measure the spacecraft inertial attitude; angle encoders to measure the relative positions of rotational sections; and a rotor-mounted star scanner to detect stars as the rotor rotates about the spin axis. Based on these sensor inputs, the onboard attitude determination system is to be designed such that it is capable of determining the absolute spacecraft attitude with respect to an inertial referenced frame.

The concept of a star-mapping technique for attitude determination aboard a spin-stabilized vehicle has been designed and successfully implemented on various Earth-orbiting satellites. They include the earlier Project Scanner Spacecraft, the Applications Technology Satellite (ATS-C), and the Orbiting Solar Observatory (OSO).[3-6] There are also numerous literature on star and inertial sensor data processing techniques for attitude estimation.[7-10] However, these are mostly for Earth-orbiting satellites and few are designed for a mission as distant and complex, and requiring as much autonomous capability, as the one for Galileo.

The Galileo spacecraft configuration is shown in Figs. 1 and 2. It consists of a spinning section (rotor), and a despun section (stator) on which is mounted an articulated scan platform to provide inertial pointing for science instruments. The spacecraft's celestial attitude is determined basically by a V-slit star scanner mounted on the rotor, while scan pointing is aided by a pair of gyros located on the scan platform.

The paper is organized as follows: In Sec. II, the attitude determination functional overview is described. Section III illustrates the schemes and techniques for processing the gyro, encoder, and star scanner data. Section IV describes the attitude determination algorithms. Section V presents the attitude determination algorithm performance and simulation. Summary and conclusions are given in Sec. VI.

II. Functional Overview

The dual-spin Galileo spacecraft requires attitude control of all three parts of the structure: rotor, stator, and scan platform. The rotor carries the high-gain antenna that must be pointed at the Earth, to within 2.1 mrad, and the thrusters whose orientation must be controlled for attitude and trajectory correction maneuvers. The scan platform carries the despun science instruments, e.g., the solid state (CCD) imaging system, that must be pointed at various targets. Stator orientation control is needed to move the scan platform and its instruments out of the exhaust plumes when firing thrusters. The clock (rotation of stator) and cone (rotation of platform) articulations are needed to provide the two degrees of freedom for scan platform pointing. In order to ac-

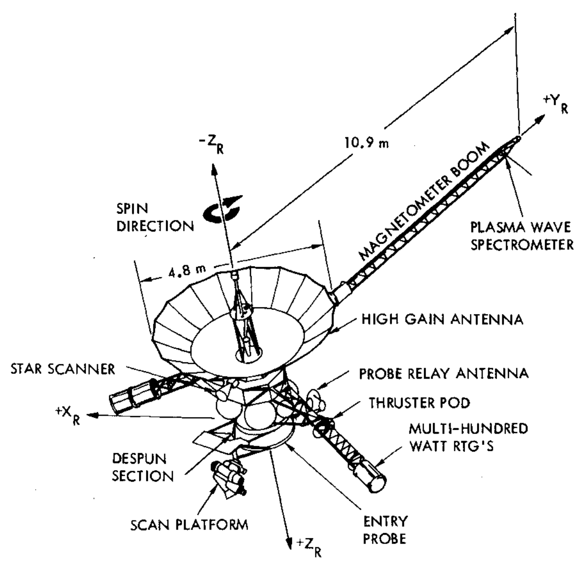

Fig. 1 Galileo spacecraft (deployed configuration).

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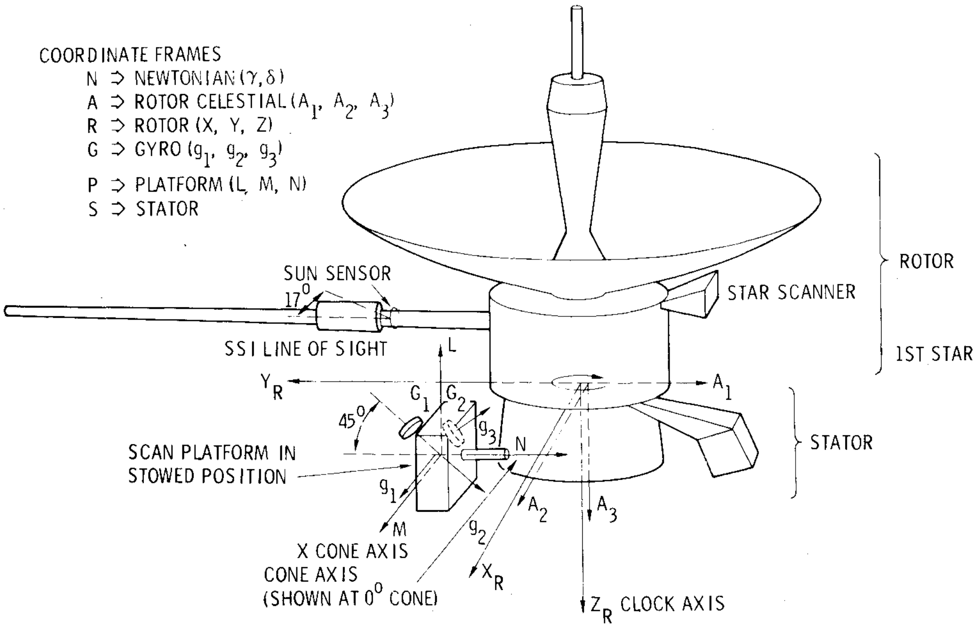

Fig. 2 Galileo coordinate systems (stowed configuration).

complish this, the attitude and articulation control system (AACS) must know the orientation of each of these spacecraft structures. This knowledge is provided by the autonomous onboard attitude determination system.

The various coordinate systems used in attitude determination, both inertially and body fixed, are shown in Fig. 2. The reference coordinate system used for all attitude determination is the Newtonian (inertially fixed) coordinate system of Earth mean equatorial of epoch 1950.0 (EME50).

The Galileo autonomous attitude determination (AD) system is composed of the following: attitude sensors, sensor data processing, and attitude estimation.

A. Sensors

1) Bendix star scanner assembly (SSA), mounted on the rotor and providing event time and intensity when its slit fields-of-view pass a star.

2) Angle encoders to measure the relative positions at the rotor-stator and stator-scan platform rotational connections (clock and cone angles respectively).

3) DRIRU II, with two 2-degrees-of-freedom dry tuned-rotor gyros mounted on the scan platform.

B. Sensor Data Processing

1) The star identification algorithms process star scanner data to provide coarse initial attitude, rotor spin rate, and a sequence of identified star-slit crossing times.

2) Encoder data processing provides the relative orientations and rates of the rotor, stator, and scan platform using estimates of the encoder angle rates.

3) The gyro outputs are integrated to maintain continuous knowledge of the scan platform orientation and rate in inertial coordinates.

C. Attitude Estimation

1) Based on star scanner data and spin rate, the sequential attitude estimator (AE) refines and maintains the inertial attitude of the angular momentum/first star coordinate system (A).

2) The inertial observer corrects the integrated gyro attitude and gyro drift rate compensation to be consistent with inertially fixed star scanner events.

The various components and their interactions are shown in the block diagram in Fig. 3.

The AD system works in two modes, defined as cruise and inertial. In the cruise mode, the AD is provided by the

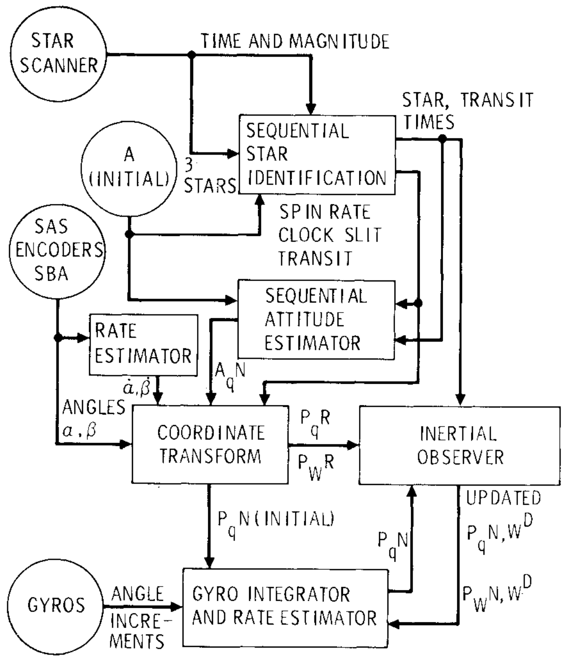

Fig. 3 Galileo attitude determination system.

sequential AE algorithm and the star data; and the rotor, stator, and scan platform positions are found using spin rate, clock encoder, and cone encoder data, respectively. The inertial mode uses the integrated gyro data (initialized from cruise mode scan platform attitude) as the primary AD data source, with the stator and rotor positions found using encoder data. The inertial observer monitors and corrects the inertial AD when star scanner data are available.

III. Attitude Determination Systems

The attitude knowledge requirement on the rotor attitude is 1.07 mrad. The knowledge requirement on scan platform pointing is 1.0 mrad when all attitude sensors are enabled. The above are all 3σ values.

This section presents the processing algorithms for all the attitude sensors data. The design of these algorithms is subject to limited onboard computer memory size (32 K words for the entire attitude control subsystem) and computation time. The primary design objective is to provide the simplest algorithms that meet the above accuracy requirements.

A. Gyro Data Processing

The inertial reference unit for Galileo provides integrated rate information about two orthogonal axes. Each gyro pulse corresponds to an angular change of 6 μrad. The raw gyro data contain errors originating from mounting misalignments, axis nonorthogonality, scale factor error, and drift, and hence must be compensated prior to its usage by the quaternion integration routine for scan platform attitude propagation.

1. Error Correction and Compensation

Let $\boldsymbol{\theta}^m$ be the vector of gyro measurements which is the integral of the rate over the gyro sampling time interval T rather than the rate components. The compensated angular increment $\boldsymbol{\theta}$ in platform coordinates is given by

$$\boldsymbol{\theta} = C\boldsymbol{\theta}^m + \boldsymbol{\omega}^d T$$

where C is a 3×3 compensation and coordinate transformation matrix whose nine parameters can be updated from the results of flight calibrations, and $\boldsymbol{\omega}^d$ is the estimated gyro drift rate.

2. Quaternion Integration Routine

Solution to the quaternion propagation equation: The differential equation governing the evolution of a quaternion $q(t)$ from EME50 to platform coordinates as a function of the angular rate quaternion

$$\omega(t) \triangleq [\boldsymbol{\omega}(t)0]^T \tag{1}$$

is given by

$$\dot{q}(t) = \tfrac{1}{2}\omega(t)q(t) \tag{2}$$

where $\boldsymbol{\omega}(t)$ is the angular rate vector of the moving platform coordinate frame with respect to EME50.

If higher derivatives of $\boldsymbol{\omega}(t)$ exist, $q(t)$ can be solved by expanding $q(t)$ in Taylor series about a quaternion estimate $q(t_0) = q_0$ evaluated at the previous sampling time t_0:

$$q(t) = \sum_{n=0}^{\infty} q_0^{(n)} T^n/n! \qquad T = t - t_0 \tag{3}$$

where the superscript (n) denotes the nth order derivative. The value of n is chosen according to the degree of desired accuracy, as illustrated in the sequel. If we define a quaternion

$$p_n(t) = q_0^{(n)}(t)q_0^*(t)$$

where $q_0^*(t)$ is the conjugate of $q_0(t)$, Eq. (3) can be rearranged into

$$q(t) = \Big(\sum_{n=0}^{\infty} p_n^{(t)} T^n/n!\Big)q_0 \tag{4}$$

We can evaluate $q(t)$ in Eq. (4) to any power of T by evaluating p_n, *i.e.*[11],

$$p_0 = [0001]^T \text{ (identity quaternion)}$$

$$p_1 = \tfrac{1}{2}\omega$$

$$p_2 = \dot{p}_1 + \tfrac{1}{2}p_1\omega = \tfrac{1}{2}\dot{\omega} + \tfrac{1}{4}\omega^2$$

etc.

Approximate quaternion solution: Obviously $q(t)$ cannot be computed directly from Eq. (4), since a direct measurement of $\omega(t)$ is not available, but the gyro output quaternion $\theta = [\boldsymbol{\theta}0]^T$, the rate integrated over a finite time interval T, is available. So a solution has to be derived which takes only θ as input, where

$$\theta = \int_0^T \omega(t)\,\mathrm{d}t = \omega_0 T + \frac{\dot{\omega}_0 T^2}{2} + \frac{\ddot{\omega}_0 T^3}{3!} + \frac{\dddot{\omega}_0 T^4}{4!} + \cdots \tag{5}$$

is obtained by expanding $\omega(t)$ in Taylor's series about $\omega(t) = \omega_0$ and integrating. Similarly, the previous two samples of the gyro output, θ^-, $\theta^=$ can be obtained by integrating over the intervals $[-T,0]$ and $[-2T, -T]$, respectively. In this way, the ω_0, $\dot{\omega}_0$, $\ddot{\omega}_0$, etc., can be evaluated in terms of gyro output directly. An mth order integration algorithm can be derived by evaluating the corresponding p_n, $n = 1,2,\ldots,m$, and retaining all the terms containing T, T^2,... up to T^m. To select an algorithm for Galileo, various typical platform motions were simulated and integrated by increasing the order of approximations. The one selected was that where an increase in order (complexity) did not give a commensurate decrease of the error. By such trading off between algorithm complexity vs algorithm truncation and roundoff errors, a fourth-order algorithm has been selected as follows:

$$q(t) = \Big\{[0001]^T + \frac{1}{2}\theta + \frac{1}{8}\theta^2 + \frac{1}{48}(\theta^3 + \theta^-\theta - \theta\theta^-) + \frac{1}{384}[\theta^4 + 4(\theta\hat{\theta} - \hat{\theta}\theta)]\Big\}q(t_0) \qquad t = t_0 + T \tag{6}$$

where $\hat{\theta} \triangleq (\theta^= - 2\theta^-)$.

3. Scan Platform Rate Estimator

The rotational rate of the scan platform coordinate frame with respect to EME50 is to be estimated using the incremental positions θ measured by the gyros mounted on the despun scan platform. Using the θ equations from above, and solving for $\omega(t)$ of order n, gives

$$\hat{\omega}_{(0)}(t) = \theta/T \qquad n = 0$$

$$\hat{\omega}_{(1)}(t) = (1/2T)(3\theta - \theta^-) \qquad n = 1$$

$$\hat{\omega}_{(2)}(t) = (1/6T)(11\theta - 7\theta^- + 2\theta^=) \qquad n = 2 \tag{7}$$

etc.

However, a pitfall to watch out for is that the resulting rate estimation accuracy is not necessarily better with increased order of n. This is due to the quantization error contained in the mesured quantities θ, θ^-, $\theta^=$, etc. For Galileo, a gyro resolution of 6 μrad and $T = 44\text{-}4/9$ ms translate into 3σ rate estimate errors of 165, 297, 450 μrad/s for estimator order of $n = 0, 1, 2$, respectively.

It is then apparent that a tradeoff between the inherent error from input quantization and the error due to inaccurate modeling (e.g., excluding terms of $\dot{\omega}(t_0)$, $\ddot{\omega}(t_0)$, etc.) is necessary. One criterion is to compare the estimated modeling error contributed from $\dot{\omega}(t_0)T + \ddot{\omega}(t_0)T^2/2 + \ldots$ with the σ error due to input quantization. If $\omega(t_0)T$, $\dot{\omega}(t_0)T^2/2$ terms dominate (i.e., high accelerations), then a higher-order estimator will be used. Conversely, if the platform rates are fairly constant, the low-order estimator is desired. For most scan platform activities of planetary spacecraft, scan platform slew rates are generally fairly constant except at the start/stop transients, where command torque profiles are to be followed. The selected scan platform rate estimator algorithm is

$$\hat{\omega}(t) = \hat{\omega}(t_0) + K_0\Big(\frac{3\theta - \theta^-}{2T} - \hat{\omega}(t_0)\Big) \tag{8}$$

during start/stop transients, and

$$\hat{\omega}(t)=\hat{\omega}(t_0)+K_1\left(\frac{\theta}{T}-\hat{\omega}(t_0)\right) \tag{9}$$

otherwise, where $\hat{\omega}(t_0)$ is the previous rate estimate. K_0 and K_1 are gains for the high- and low-order estimators, respectively, which provide some filtering for noise reduction.

B. Encoder Data Processing

The relative positions of three movable parts of the Galileo spacecraft are measured by angle encoders located on the articulation axes. For the dual-spin configuration of Galileo, the connection between the spun rotor and the stationary stator is the spin bearing assembly (SBA). Their relative rotation is measured by a 16-bit optical angle encoder in the SBA. This encoder is called the "clock" encoder, and the rotation of the stator with respect to the rotor is the clock angle (α).

The scan platform, holding science instruments including the solid state imaging (SSI) instrument, is articulated with respect to the stator by the scan actuator subsystem (SAS), which contains the 16-bit optical "cone" angle encoder. The cone (β) angle is the angle of rotation of the SSI line-of-sight from the high gain antenna boresight (normally pointed at Earth).

In determining the relative positions of the rotor and scan platform, not only the two encoder angles must be considered, but also the associated errors and structural misalignments that are constant and can be measured before launch or estimated in-flight using spacecraft based data. These errors are the encoder null offsets ($\tilde{\alpha}$ and $\tilde{\beta}$), nonorthogonality of the clock and cones axes of rotation (ϵ_1), and nonorthogonality of the SSI line-of-sight with the cone axis (ϵ_2). The coordinate transformation from the rotor coordinate system (X,Y,Z) to the scan platform coordinate system (MNL) is then composed of a series of single-axis rotation transformations (see Fig. 2 for the nominal coordinate system definitions). Whether these coordinate transformations are represented by direction cosine matrices or quaternions, they are represented symbolically by

$$^{P}T^{R}=[\epsilon_2]_1[\pi+\beta+\tilde{\beta}]_2[\epsilon_1]_1[\alpha+\tilde{\alpha}]_3 \tag{10}$$

where $[\xi]_i$ represents a right-handed rotation/transformation of angle ξ about the ith coordinate axis and the order of rotations is right to left (α first, ϵ_2 last).

The encoder-read process will convert the 16-bit integer encoder values to floating point radians (range 0-2π) and add the null offset correction. The angle rate is not directly available and must be calculated from the encoder angle values by the encoder rate estimation algorithm. This algorithm is (same for both α and β): At the ith computation cycle, $\Delta\beta_i=\beta_i-\beta_{i-1}$. To correct for encoder rollovers, if $\Delta\beta_i<-\pi$, then $\Delta\beta_i=\Delta\beta_i+2\pi$; if $\Delta\beta_i>+\pi$, then $\Delta\beta_i=\Delta\beta_i-2\pi$. The estimated rate is then

$$\dot{\beta}_i=\dot{\beta}_{i-1}+G(\Delta\beta_i/\Delta t-\dot{\beta}_{i-1}) \tag{11}$$

where G is the smoothing factor.

The encoder angle resolution of 95.87 μrad and the 44-4/9 ms cycle time (Δt) give an unsmoothed ($G=1.0$) rate estimate error of 0.88 μrad/s, 1σ. Smoothing the estimate with $G=0.2$ reduces the error to 0.13 μrad/s, 1σ.

The computations of the rate of the scan platform with respect to the rotor, expressed as a vector in scan platform coordinates is represented by

$$^{P}\omega^{R}=[\epsilon_2]_1\left\{\begin{bmatrix}0\\ \dot{\beta}\\ 0\end{bmatrix}+[\pi+\beta+\tilde{\beta}]_2[\epsilon_1]_1\begin{bmatrix}0\\ 0\\ \dot{\alpha}\end{bmatrix}\right\} \tag{12}$$

Some hardware peculiarities of the encoder must be considered. The encoder value is returned when a torque command is sent to the axis torque motor; however, the value was sampled shortly after the *previous* torque command. The encoder-read procedure will attach a time-tag to the data representing the time at which it was sampled, not when it was read.

C. Star Scanner Data Processing

The processing of the star scanner (SSA) output is by the star identification algorithms which were described in an earlier paper.[12] This process will be described briefly here.

The rotor-mounted star scanner has two optical slits (one vertical and one tilted) in front of a photomultiplier. As the rotor spins, the slits scan the sky and when a star passes through a slit field-of-view the photomultiplier emits a pulse and the SSA electronics outputs the pulse magnitude, time of the pulse trailing edge, and an interrupt to the AACS computer. These data are processed by the star identification (SID) algorithm.

The SID is provided a set of three stars to be tracked and an initial estimate of the angular momentum vector from which it computes the initial (coarse) inertial attitude of the angular momentum—Star 1 coordinate system (A) which is refined and maintained by the sequential attitude estimation algorithm. The SID then searches for and locks on the pulses from the three stars. This is the beginning of the whole onboard attitude determination process.

After this initialization, the SID algorithm selects from the sequence of star scanner pulses the pairs that correspond in magnitude, time, and time differences to what is expected from the three stars currently selected for attitude determination and estimates the rotor spin rate. The data provided to the attitude determination algorithms are the rotor spin rate, and for each identified pulse pair, the two pulse times and the identity of the star crossing the slits.

IV. Attitude Determination

A. Attitude Determination Observations

The observations used for attitude determination represent the difference between the current estimate of spacecraft attitude and the actual attitude. This difference is used to determine an attitude estimate correction. For Galileo, the stars, fixed in inertial coordinates, are the absolute reference and the star scanner slit crossing events are the basis for at-

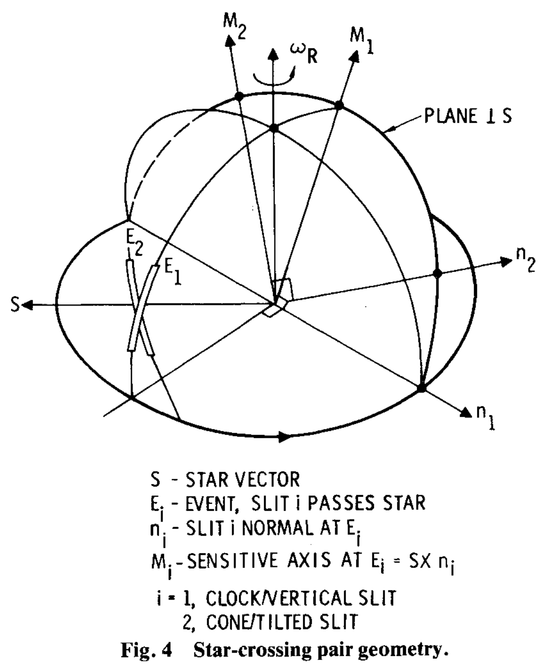

Fig. 4 Star-crossing pair geometry.

titude determination. At a slit crossing, the star is in the plane of the slit, thus the star direction vector is orthogonal to the vector normal to the slit plane. The geometry of a star crossing pair is shown in Fig. 4. The slit normals are known in rotor coordinates from the spacecraft design and refined by in-flight calibration.

The observation is the dot product of the star vector (S) and the slit normal (n) as computed using the current attitude estimate. If there were no error, the value would be zero. This scalar observation is sensitive to attitude errors affecting the star-normal angular separation, i.e., the component of attitude error (γ) along the normal to the star-normal plane. These relationships give the observation equation that is to be used in estimating the attitude correction.

$$S \cdot n = (S \times n) \cdot V + \text{noise}$$

The noise in the equation determines the quality of the observation and is assumed to be zero mean, random, and uncorrelated between observations, since any systematic errors are removed by in-flight calibration. Sources of noises are such as star crossing timing error, spacecraft rate estimate error, encoder quantization, as well as computational errors (e.g., approximations and roundoff).

B. Sequential Attitude Estimator

The attitude estimator (AE) for the Galileo spacecraft is designed to sequentially process the star scanner measurements and determine the spacecraft's attitude and angular momentum vector in EME50 for the purpose of high gain antenna pointing control, and determination of rotor and scan platform attitudes during cruise mode when gyros are off. The AE formulation uses the least-squares method to determine the attitude which satisfies the necessary conditions of star and slit normal orthogonality.

1. Formulation of the Dynamical Model

A spacecraft inertial coordinate frame A established by three orthonormal vectors,

$$A = [a_1, a_2, a_3]$$

is first defined, with a_3 aligned with the estimated angular momentum vector H, and a_1 lying in the plane defined by H and the line-of-sight (LOS) of the "first" reference star.

The basic concept in the model formulation is that when a star falls in the scanner slit field-of-view (FOV), the star vector S is orthogonal to a vector n normal to the scanner slit. The star vector is defined in EME coordinates (N), and the slit normal is defined in rotor coordinates (R).

To determine the coordinate frame A, it is necessary that both S and n are expressed in this coordinate frame, i.e.,

$$S = A^T S_N \tag{13}$$

and

$$n_j = [\omega(t_0 - t_j)]_3 n_{jR} \qquad j = 1,2 \tag{14}$$

where ω is the rotor spin rate; n_1 and n_2 are the clock and cone slit normal vectors, respectively; t_1 and t_2 are the clock and cone transit times, respectively; t_0 is the clock slit transit time of the "first" referenced star; and n_{jR}, $j=1,2$ are the clock and cone slit normals in R, which are constant vectors for a known slit mounting orientation after in-flight misalignments calibration.

With the presence of a passive nutation damper and an onboard wobble control algorithm, nutation and wobble are insignificant during steady-state operations. Furthermore, onboard memory size and execution time margins are major concerns in the design of control algorithms. In the sequel, we will consider a pure-spin dynamics model for the AE formulation. The slit normal is expressed in a coordinate frame which differs slightly from A. This small difference is caused by the effect of nutation and wobble, and is regarded as error in the estimator model. Nutation will be averaged out over time. Wobble, although small, is synchronized with the star transits, and may or may not be averaged out depending on the star distribution.

2. Attitude Estimator Algorithm

At each star crossing time k, our objective is to determine $A(k)$ from the previous estimate $A(k-1)$ from the star vector $S(k)$, and the transit times $t_1(k)$ and $t_2(k)$ provided by the sequential SID. Let γ be the desired angular rotation (attitude error) about the axes of $A(k-1)$ to assume a new $A(k)$. By small angle approximation, we formulate the following equation:

$$\{[I - \gamma\times]S(k)\} \cdot n_i(k) = v_i(k) \qquad i = 1,2 \tag{15}$$

where $S(k)$ and $n_i(k)$ are expressed in the $A(k-1)$ coordinates, and $v_i(k)$ is the associated measurement noise. The notation $[e\times]$ is a matrix cross-product operator defined as

$$e\times = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix}, \text{ for } e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Rearranging Eq. (15), one can obtain

$$\epsilon_i(k) = (S(k) \times n_i(k)) \cdot \gamma(k) + v_i(k), \qquad i = 1,2 \tag{16}$$

where $\epsilon_i(k) = S(k) \cdot n_i(k)$. A coordinate system, L, is defined with the star vector aligned with L_1, and L_2 aligned with the clock slit normal at transit. This enables simple expressions of the sensitive axes' ($S \times n_i$, $i=1,2$) orientations to be defined in the L_2-L_3 plane (Fig. 5). As the change in the actual attitude between the two transits $t_1(k)$ and $t_2(k)$ is small, the attitude update can be performed immediately after the cone slit transit, using both clock and cone transit information. With $\gamma(k)$ expressed in L_2-L_3 coordinates, Eq. (16) can be written as

$$\epsilon(k) = \begin{bmatrix} 0 & 1 \\ -\sin\zeta & \cos\zeta \end{bmatrix} \gamma(k) + V(k)$$

$$\zeta \text{ (con slit tilt)} \neq 0 \qquad \epsilon = [\epsilon_1, \epsilon_2]^T \qquad V = [v_1, v_2]^T \tag{17}$$

With the assumption that the error γ is much larger than the noise V, Eq. (17) is solved to give

$$\gamma(k) = [g_1, g_2]\epsilon(k) \triangleq \begin{bmatrix} \cot\zeta & -\dfrac{1}{\sin\zeta} \\ 1 & 0 \end{bmatrix} \epsilon(k) \tag{18}$$

in the L_2-L_3 plane. Furthermore, g_1, g_2 can also be expressed in terms of n_1 and n_2 at their respective times of star transit. From Fig. 5,

$$g_1 = \frac{1}{\sin\zeta} n_2 \qquad g_2 = -\frac{1}{\sin\zeta} n_1 \tag{19}$$

Then, with n_1, n_2 defined in $A(k-1)$, $\gamma(k)$ can be transformed back to the $A(k-1)$ frame by

$$\gamma(k) = [\epsilon_1(k) n_2(k) - \epsilon_2(k) n_1(k)]/\sin\zeta \tag{20}$$

Although we have derived a desired rotation about the coordinates of $A(k-1)$ to assume a new $A(k)$, it is desirable, for better noise tolerance, that the actual attitude update is controlled by different estimator gains $K(k)$ depending on whether the spacecraft is in the transient or steady state. Hence the actual rotational angle is

$$\rho(k) = K(k)\gamma(k)$$

At transient state, fast reduction in the attitude error is desirable and hence full gain on star information is used. Such a gain, however, weighs equally on the true attitude error as much as on the effect of noise. At steady state, the gain is reduced to a minimum magnitude (e.g., comparable to the size of jittering), so that sporadic, high-frequency noise can be adequately filtered out.

The estimator gain $K(k)$ can be set at

$$K(k) = \begin{cases} 1 & k \leq k_0 \text{ transient state} \\ K(k-1)/[\mathrm{GF} + K(k-1)] & k > k_0 \text{ steady state} \\ & k = 1,2,3,\ldots \end{cases} \quad (21)$$

where k is a star crossing counter (three per revolution), and is set to zero whenever an external torque is applied to the spacecraft (e.g., thruster firings). The gain factor (GF) is 1 minus the desired steady-state value of K. k_0 can be chosen based on the distribution of the three identified stars S_i, $i=1,3$ in the inertial space. In a deterministic sense, the estimation error $e(k)$ can be reduced $e(k) = \lambda e(k-3)$ per revolution in the transient state, where

$$\lambda = |(S_1 \cdot S_2)(S_1 \cdot S_3)(S_2 \cdot S_3)| \quad 0 \leq \lambda < 1 \quad (22)$$

is a star distribution index indicating the degree of orthogonality in the referenced stars' distribution. Since the spacecraft attitude is initialized with an error around 17 mrad, to reduce the error to 0.3 mrad, the value of k_0 can be approximated by

$$k_0 = 3\ln(0.3/17)/\ln\lambda \simeq 3 + 80\lambda^2 \quad (23)$$

The attitude A is then updated by

$$A(k) = A(k-1)(I + \rho(k)\times) \quad (24)$$

C. Inertial Observer

The function of the inertial observer is to correct the scan platform attitude determined from integrated gyro data. It corrects not only position offsets such as initialization error but also the gyro drift rate compensation to permit continued, accurate integration if the star scanner data stops, as during a spacecraft maneuver or in a high radiation period in the vicinity of Jupiter.

When the star identification algorithm has selected a star crossing pair, the inertial observer executes using the following data from various sources: t_1, t_2—two star crossing

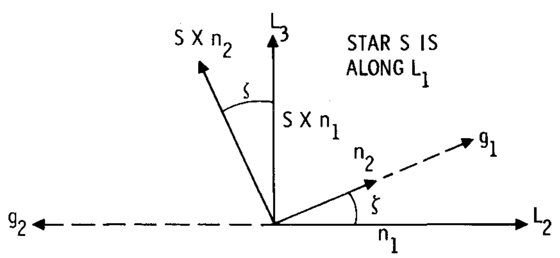

Fig. 5 Star-normals coordinate frame (*L*).

times; S—star direction vector in EME50; ${}^Pq^N$, ${}^P\omega^N$, t_g—scan platform attitude quaternion and rate with respect to EME50 at time t_g; ${}^Pq^R$, ${}^P\omega^R$, t_e—scan platform with respect to rotor, quaternion and rate based on encoder data at t_e; n_1, n_2—slit normals in rotor coordinates.

The coordinate system chosen to compute the observation equation is the scan platform coordinate system at time t_g, since this is the one to be updated. The star vector is mapped into this coordinate system by

$$S_P = {}^Pq^N \;\; S \;\; {}^Nq^P \quad (25)$$

For each observation $(i=1,2)$ the direction of the slit normal n_i at time t_i must be found based on the available data. To do this, an assumption of constant rate must be made. The quaternion from the rotor at $t_i(R_i)$ to scan platform is found as follows:

$${}^Pq^Ri = Q[({}^P\omega^N - {}^P\omega^R)(t_e - t_i)]Q[{}^P\omega^N(t_g - t_e)]{}^Pq^R \quad (26)$$

where $Q[\xi]$ represents the quaternion corresponding to a vector rotation ξ. The slit normal is then mapped to platform coordinates

$$n_{iP} = {}^Pq^Ri n_i {}^Ri q^P \quad (27)$$

The observation equations used are the same as for the sequential AE,

$$\epsilon_i = S_P \times n_{ip} \cdot \gamma + v_i \qquad i = 1,2 \quad (28)$$

where

$$\epsilon_i = S_P \cdot n_{iP}$$

The inertial observer does not have to be reinitialized frequently, since the gyros sense and track any attitude changes such as reorientation of the angular momentum vector to track the Earth. The estimator is then optimized for steady-state accuracy rather than rapid convergence by assuming that the attitude error variance is small compared to the noise variance.

The attitude error estimator that results is

$$\gamma = S_P \times (n_{1P}\epsilon_1 + n_{2P}\epsilon_2) \quad (29)$$

and the attitude and drift rate (ω^d) updates are

$${}^Pq^N = Q[K_p\gamma]{}^Pq^N \quad (30)$$

$$\omega^d = \omega^d + K_r\gamma \quad (31)$$

where K_p is the constant position gain and K_r is the constant rate gain.

To improve convergence, a period of higher K_p value, with $K_r = 0$, is used to remove attitude initialization errors without adversely affecting the drift rate correction. This mode is used whenever the gyro integrator is initialized or there has been a period of several hours without star scanner data.

The assumption of constant rate required in the process of data synchronization extrapolation is not valid during thruster firings or scan platform start/stop slewing transients. Processing of star data during these times is inhibited to prevent errors caused by unmodeled angular acceleration.

D. In-Flight Calibration

The accuracy of the onboard attitude determination is affected by many constant error sources. If the accuracy is to approach the theoretical accuracy based on random data noise, these sources of systematic errors must be removed. This is done by in-flight calibration, meaning that actual data from the spacecraft are processed to estimate these error sources. The processing is done on the ground and the resulting values of onboard model parameters are transmitted to the spacecraft to be used in correcting data onboard.

Examples of systematic errors are star scanner slit alignment with respect to the rotor; SBA and SAS encoder null offsets and nonlinearity; mechanical alignments of SBA, SAS, SSI instrument; and gyro scale factors, alignment, and drift rate.

Taking the in-flight calibration as a whole (though some subsets of the errors may be calibrated separately) the data needed from the spacecraft are star crossing events, encoder angles, raw gyro outputs, and star pictures using the SSI camera. While collecting these data, the spacecraft is commanded to go through a sequence of motions to change the geometrical relationships between the error sources. The ground processing is an estimation process using a complete spacecraft state model and all the error sources as parameters to be estimated. The resulting values are put into the form used in onboard error correction and loaded into the AD subsystem computer memory.

V. Algorithm Performance and Simulation Results

The AE was simulated with various orientations of the angular momentum (*H*) vectors in EME50. Transit time noise of $\sigma=0.5$ and 1.3 ms, nutation angles up to 2 mrad, and distribution index λ from 0.02 to 0.76 were tested. Convergence in *H* to below 0.5 mrad error was achieved in all test cases, including some others with initial errors as large as 10 deg. Figure 6 compares the convergence of the *H* vector for two different star distributions $\lambda=0.08$, 0.76 under a noise-free situation. Note, however, that large λ is quite unlikely, as small values of λ are one of the major criteria for selecting the three stars to be used. Figure 7 depicts the estimation error in *H* vs star observations in the presence of noise. The error in quaternion integrator algorithm drifts at 0.04 mrad/h (3σ) with the spacecraft in precession. The integrator is executed once every 44-4/9 ms. The scan platform rate estimation error is normally 160 μrad/s (3σ) and is 310 μrad/s (3σ) during start/stop transients of slews.

The performance of the inertial observer algorithm has been evaluated using a combination of simulation and linear analysis. The linear model was used to predict the response for long periods of time, and to do covariance propagation for statistical error analysis.

The accuracy of the inertial observer attitude and drift rate estimate depends on the spatial distribution of the three stars being used as well as the noise in the measurements. If the stars are closely aligned, star distribution index (λ) is high, the component of error along the mean star line is not well determined. A "worst case" star distribution ($\lambda=0.8$) increases the uncertainty along that line by as much as a factor of 5. The measurement noises (3σ random error) used in worst case performance prediction are:

Star slit crossing time	3.6 ms
Encoder quantization	96 μrad
Encoder rate estimation	400 μrad/s
Gyro quantization	6 μrad
Gyro rate estimation	255 μrad/s
Gyro drift rate stability	0.72 μrad/s/8 h
Star distribution factor	$\lambda=0.8$

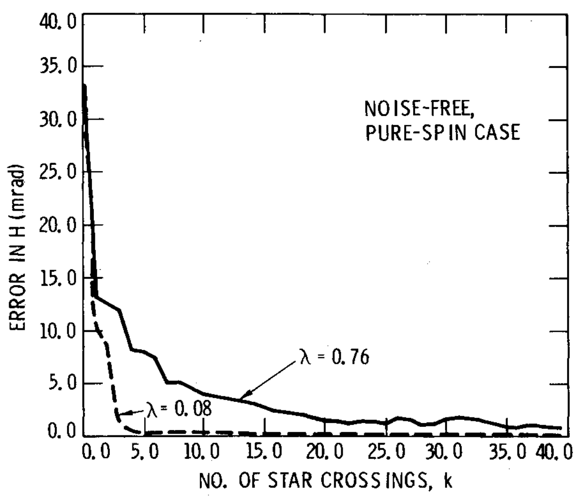

Fig. 6 Angular momentum (*H*) attitude error.

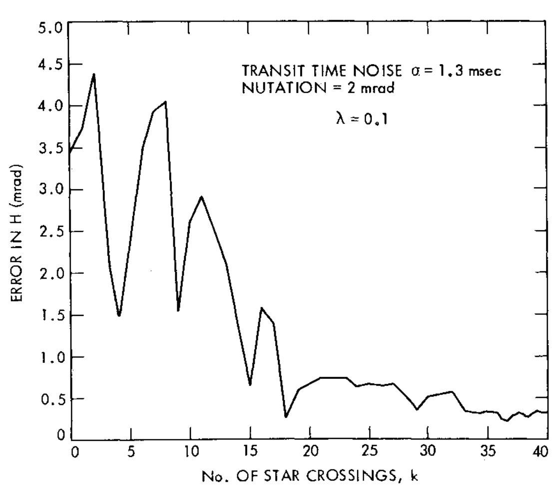

Fig. 7 Angular momentum (*H*) attitude error.

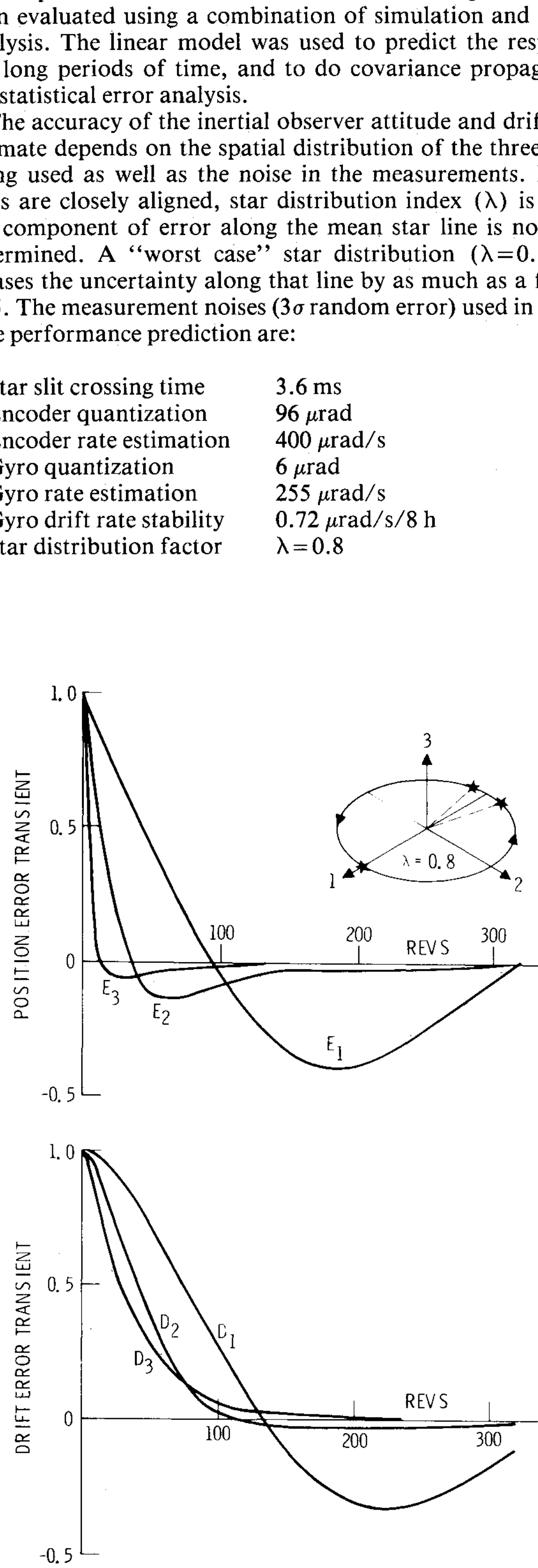

Fig. 8 Inertial observer response with bad star distribution.

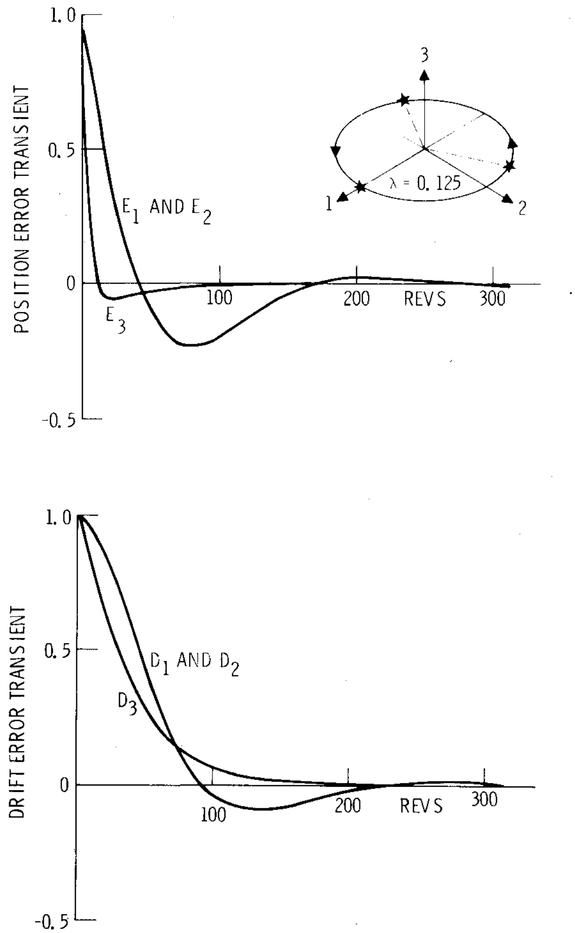

Fig. 9 Inertial observer response with good star distribution.

These noise values result in a "worst component" steady-state error of 0.43 mrad (3σ) in position and 0.32 μrad/s (3σ) in drift rate. The error with nominal values rather than worst cases are about an order of magnitude better. Figures 8 and 9 show the inertial observer position and drift error transient responses at the low steady-state gain for $\lambda = 0.8$ and $\lambda = 0.125$, respectively.

VI. Summary and Conclusion

An onboard inertial attitude determination (AD) system for the dual-spin Galileo planetary spacecraft has been presented. Methods of processing the output data from the AD sensors—the gyros, encoders, and star scanner—are illustrated. These processed sensor outputs are used for determining basically two attitude coordinate systems. The first is the scan platform coordinates, whose relative attitude is propagated by a quaternion integration algorithm using compensated (in drift and misalignments) gyro outputs, and whose absolute attitude is periodically determined by an inertial observer using the star scanner data. The second is an inertial spacecraft coordinate frame having one axis aligned with the angular momentum vector, and is determined by a least-squares sequential estimator based on the star scanner slit geometry and star transit time history.

Algorithm performance evaluated by computer simulation has been presented. The inertial observer provides a worst case steady-state error of 0.43 mrad (3σ) in platform position and 0.32 μrad/s (3σ) in drift rate. The quaternion integration algorithm propagates the scan platform attitude with an error drift of 0.04 mrad/h (3σ) due to truncation and roundoffs. In the presence of nominal nutation and transit time noise, the sequential estimator determines the spacecraft rotor attitude within an accuracy of 0.5 mrad (3σ). Based on these simulation results, the attitude determination requirements for the Galileo mission will be satisfactorily met.

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